

(b) If $y = (\sin^{-1} x)^2$, then prove that

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0.$$

8. (a) Evaluate :

$$\int_0^1 \frac{(\tan^{-1} x)^2}{1 + x^2} dx$$

(b) If $\phi(n) = \int_0^{\pi/4} \tan^n x dx$, then prove that :

$$\phi(n) + \phi(n-2) = \frac{1}{n-1}$$

9. (a) Show that the four points with Position vectors

$$(4\hat{i} + 5\hat{j} + \hat{k}), (-\hat{j} - \hat{k}), (3\hat{i} + 9\hat{j} + 4\hat{k}) \quad \text{and}$$

$$4(-\hat{i} + \hat{j} + \hat{k}) \text{ are coplanar.}$$

(b) If $\vec{a}, \vec{b}, \vec{c}$ be any three vectors, then prove that :

$$\begin{bmatrix} \vec{a} + \vec{b} & \vec{b} + \vec{c} & \vec{c} + \vec{a} \end{bmatrix} = 2 \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$$

Roll No.

BCA-105(N)

B. C. A. (First Semester) EXAMINATION, Dec., 2017

(New Course)

Paper Fifth

MATHEMATICS—I

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt questions from all Sections as directed.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

(Short Answer Type Questions)

Note : Attempt all questions from this Section. Each question carries 3 marks.

1. (A) Using properties of determinants, show that :

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

(B) If matrix :

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}$$

find $3A - 2B$.

- (C) If $\lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80$, find the value of n .
- (D) Differentiate the following function w. r. t. x ,
 $e^{2x} \log 3x$.
- (E) Write the statement of Lagrange's mean value theorem with suitable example.
- (F) Evaluate the integral :

$$\int \sin^4 x \cos^3 x \, dx$$

- (G) Write the definition of Gamma function with its properties.

- (H) If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, find the value of $\left(\vec{a} + 3\vec{b} \right) \cdot \left(2\vec{a} - \vec{b} \right)$.

- (I) If $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{k}$, find $\left| \vec{b} \times 2\vec{a} \right|$.

Section—B

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

2. Find the adjoint and inverse of the matrix :

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 6 & 2 \\ -3 & 2 & -1 \end{bmatrix}$$

3. Find the characteristic equation of the matrix :

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

and verify that it is satisfied by A .

4. Describe the different kinds of discontinuities of a function and show that the function defined by :

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

is continuous at $x = 3$.

5. Apply Maclaurin's theorem to find the expansion in ascending powers of x of $\log_e (1 + e^x)$ upto the term containing x^4 .

Section—C

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

6. (a) Show that $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$ does not exist.

- (b) Investigate the maxima and minima of :

$$f(x) = 2x^3 - 21x^2 + 36x - 20$$

7. (a) Verify Rolle's theorem for the function :

$$f(x) = x^3 - 12x, \quad x \in [0, 2\sqrt{3}]$$